Perturbed cross-flow boundary layer: nontrivial effects of the obliquity angle at small and high reynolds numbers

PERTURBED CROSS-FLOW BOUNDARY LAYER: NONTRIVIAL EFFECTS OF THE OBLIQUITY ANGLE AT SMALL AND HIGH REYNOLDS NUMBERS

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Abstract We carry out an exploratory linear study on the transient dynamics of arbitrary three-dimensional perturbations acting on the Falkner-Skan-Cooke cross-flow boundary layer. The analysis is in particular focused on the effects of the direction of the perturbations at low and high Reynolds numbers. We show evidence of a non-trivial dependency of the transient and asymptotic behavior on the travelling waves obliquity with respect to the base flow.

The Falkner-Skan-Cooke cross flow boundary layer stability has been analyzed in literature using modal theory [1], in the context of receptivity and transient optimal perturbations [2, 3] and experimentally [4]. Here we treat the full linear three-dimensional perturbation problem by paying particular attention to the role of the obliquity of the perturbation with respect to base flow direction. In literature there are few contributions with a specific focus on the effect of the angle of obliquity on the perturbation transient life. Works on optimal perturbations [3] usually consider obliquity angles in the neighborhood of a fixed value. For instance, Breuer and Kuraishi [2] investigated the direction of the waves, but varying simultaneously ϕ and k. The exploratory analysis here proposed highlights a rich and, for certain aspects, counterintuitive scenario on the role of the perturbation direction. We observed that the "instability" of the waves does not depend in a trivial way from their obliquity: there are in fact configurations where both the longitudinal and orthogonal waves are unstable while oblique ones in-between are stable.

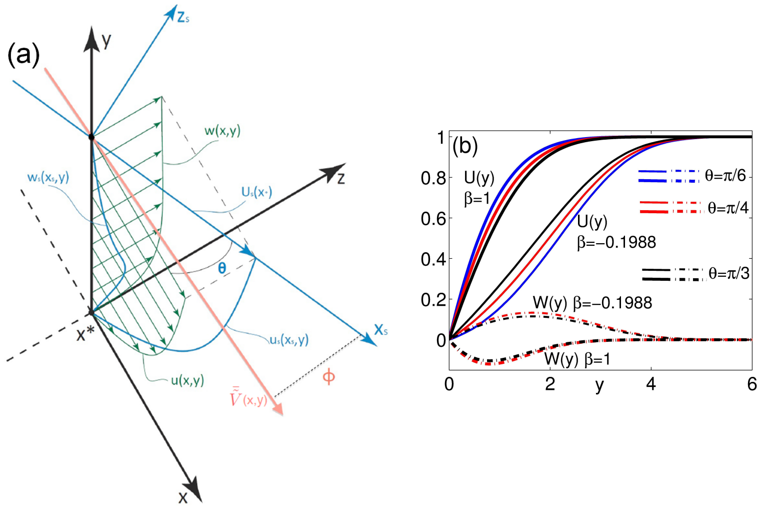

Figure 1. (a) Reference system and sketch of the cross-flow boundary layer: θ is the angle of cross-flow, ϕ is the angle of obliquity. (b) Components of the base flow, $U(y)$, $W(y)$, $\beta = -0.1988, 1$ and $\theta = \pi/6, \pi/4, \pi/3$.

We consider sub- and supercritical flow configurations (the Reynolds number, based on the displacement thickness δ^*, is equal to 100 and 5000), subject to adverse and favorable pressure gradients (Hartree parameter, β, equal to -0.1988 and 1, respectively). The cross flow angle, θ, between the streamwise direction and the chordwise direction, is taken equal to $\pi/6, \pi/4, \pi/3$ (see Figure 1). Concerning the direction and wavelength of the perturbation, we vary both the obliquity angle with respect to the streamwise direction and the polar wavenumber, ϕ and k. The transient and asymptotic behavior of the perturbative waves is observed through the amplification factor, $G(t)$ (defined as the kinetic energy density, $E(t)$, normalized over its initial value), the temporal growth rate, $r = \log(E(t))/2t$, and the frequency, ω. This last is defined as the temporal derivative of the phase at a fixed distance from the wall.

Fig. 2 shows the effect of the obliquity angle over two very different values of the Reynolds number. The amplification factor, $G(t)$, is shown in the top panels, the frequency, $\omega(t)$, in the bottom panels. Panel (a) contains the first result that we wish to highlight here. One can see that even at a Re value as low as 100, in the presence of a sufficiently high pressure gradient, a small range of unstable longitudinal wavenumbers still exist: the longitudinal wave is unstable, while the oblique and the orthogonal waves have an initial decay followed by a transient growth and a final decay. Panels (b) and (c) present the case with the higher Reynolds number, 5000. Here, both a favorable and an adverse pressure gradient are considered together with a cross flow angle of $\pi/6$. In the favorable situation, the orthogonal wave is unstable while the longitudinal and oblique perturbations are asymptotically stable, but have a growth-decay phase in the early transient. In panel (c), the most unstable configuration achievable with our choice of parameters, we see that the longitudinal and the orthogonal waves are unstable but oblique waves in between are not, and this is unexpected. Because, usually, if one sees

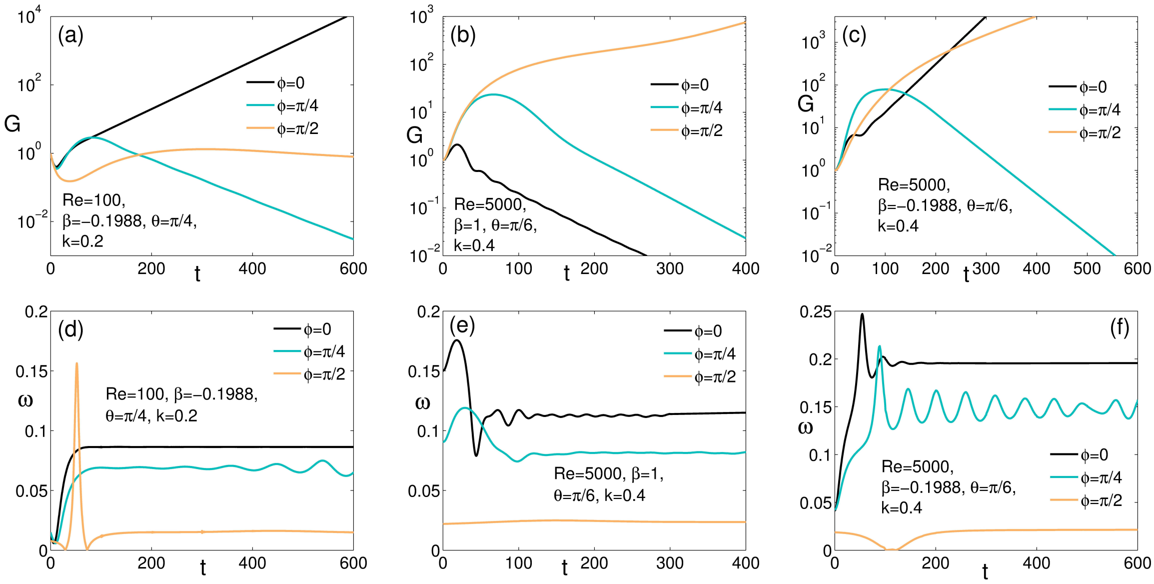

Figure 2. Role of the obliquity angle, $\phi = 0.\pi/4, \pi/2$. Temporal evolution of the amplification factor, G(t) (panels a, b, c), and the frequency, $\omega(t)$ (panels d, e, f). (a-d) $Re = 100$, $\beta = -0.1988$, $\theta = \pi/4$, $k = 0.2$. (b-e) $Re = 5000$, $\beta = 1$, $\theta = \pi/6$, $k = 0.4$. (c-f) $Re = 5000$, $\beta = -0.1988$, $\theta = \pi/6$, $k = 0.4$.

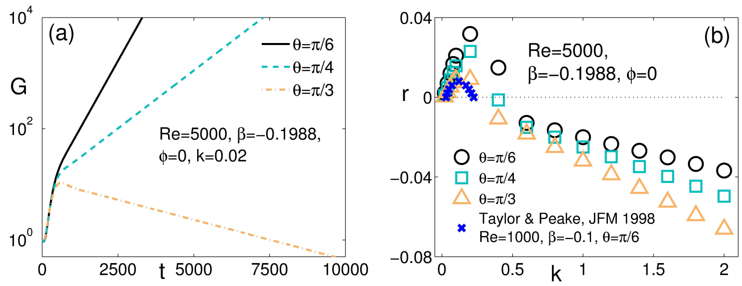

Figure 3. Role of θ: Temporal evolution of the amplification factor, $G(t)$ (panel a), and the temporal growth rate $r(k)$ in the asymptotic limit (panel b), $\beta = -0.1988$, $Re = 5000$, $\phi = 0$. Pinch points results by Taylor and Peake [1] are added for a qualitative comparison.

instability in the longitudinal direction, one then sees a progressive tendency to stability toward the orthogonal direction, and viceversa. Instead, here intermediate angles, as $\phi = \pi/4$, have an intense initial growth and then become stable.

Independently of the stable or unstable asymptotic behavior of the perturbation, the frequency along the transient present sudden impulsive variations, which we may describe as frequency jumps (see panels d-e-f). We interpret these jumps as the transition between the early part of the transient and the beginning of an intermediate term that show up for times large enough for the influence of the fine details of the initial condition to disappear.

The combined effect of β and θ has been investigated by Taylor and Peake [1]. They found that asymptotically the pinch points with an adverse pressure gradient flow are more unstable at lower cross flow angles, while for negative pressure gradient the opposite is true. In Fig. 3, we show that this result is valid not only for the pinch point formulation but, more in general, for an arbitrary variation of the perturbation wavelength. A comparison with available data from [1] is presented in panel (b). Even if the Reynolds number is different (1000 versus 5000) the comparison is good.

We have obtained these results by adopting the initial-value problem formulation in the velocity-vorticity formulation. We perform a Laplace-Fourier transformation of the governing disturbance equations in the streamwise and spanwise directions, and then numerically solve the resulting partial differential equations [5, 6]. This approach offers an alternative means for which arbitrary initial conditions are imposed and their full temporal behavior, including both the early transient and the long-time asymptotics, can be observed. As initial condition we consider a Gaussian function, $v(t = 0, y) = y^2 \exp(-y.^2)$, $\omega_y(t = 0, y) = 0$. Such initial condition concentrates the energy of the wave in the shear region [7].

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